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Nonlinear growth of magnetic islands by passing fast ions in NSTX

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Outline

- Introduction
- Theoretical model
- Experimental setup
- Comparison of simulation and experiment
- Discussion and conclusion

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Do fast ions affect magnetic island growth in NSTX?

- Fusion products in reactor relevant plasma may affect NTM stability
- Study of fast ion effect on NTM requires comprehensive set of measurements
  - TRANSP can provide time-dependent thermal and fast ion profiles self-consistently
- We show that passing fast ions can open gate for magnetic island growth in NSTX
- We assume single fluid model and fit modified Rutherford equation coefficients
  - Kinetic neoclassical polarization current theory [1] is developed from single fluid model
  - Modified Rutherford equation coefficients represent measurement uncertainties

[1] Cai, Nucl. Fusion **56** 126016 (2016)

Modified Rutherford equation governs island growth physics

- Tearing mode stability index [1]: Free energy within current density profile
- Neoclassical drive term [1]: Drive caused by loss of bootstrap current
  - Correction considering electron cross field transport [2]
- Polarization current stability term [3]: Subtlety involving polarization current
  - Toroidal current with zero surface average that *may* stabilize and create the “gate”
  - Conceptually explains island saturation but difficult to validate experimentally
- Curvature stabilization term [4]

[1] Fredrickson *et al.*, Phys. Plasmas **7** 4112 (2000)
[2] Fitzpatrick, Phys. Plasmas **2** 825 (1995)
[3] Wilson *et al.*, Phys. Plasmas **3** 248 (1996)
[4] Gorelenkov *et al.*, Phys. Plasmas **3** 3379 (1996)

$$\frac{1}{k_r}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_b\left[\frac{16 J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2}\right] - k_p\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_c\frac{\beta_\theta \varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

Neoclassical polarization current is non-negligible for fast ions

- Parallel current may form to replenish lost bootstrap current [1]
  - Effectively neutral beam current drive
  - Weakened if poloidal fast ion Larmor radius is larger than the magnetic island (≈ orbit loss)
- Kinetic neoclassical polarization current is recently suggested [2]
  - Fast ion equivalent of neoclassical polarization current
  - Loss of ion *E*×*B* drift leads to toroidal current that restores charge neutrality
  - Takes effect after the formation of island separatrix – Not a trigger mechanism!
  - Introduces ion density profile into magnetic island physics

[1] Hegna and Bhattacharjee, Phys. Rev. Lett. **63** 2056 (1989)
[2] Cai, Nucl. Fusion **56** 126016 (2016)

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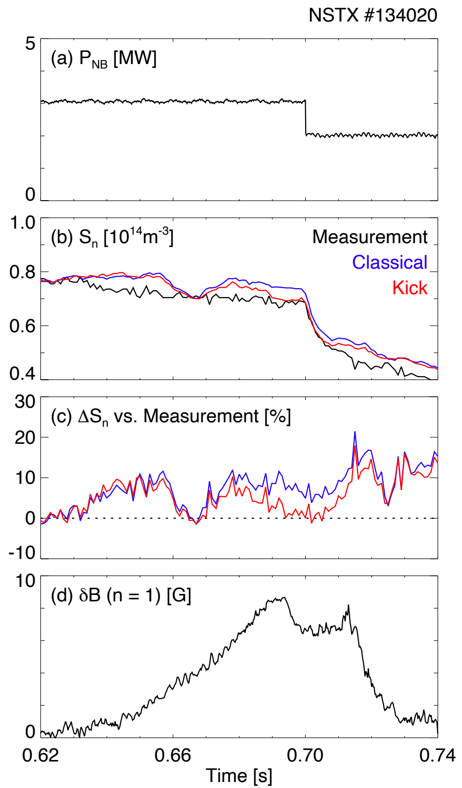
Kick model is used to evaluate MHD induced fast ion transport

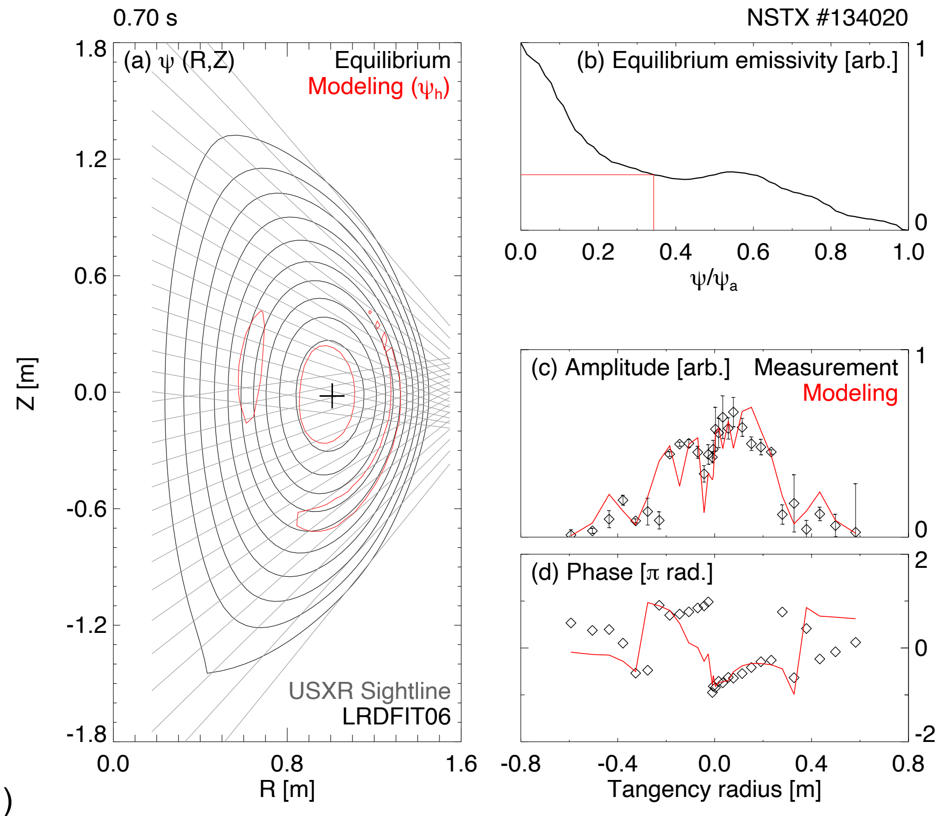
- NSTX #134020 is selected for analysis
  - Neutral beam heated H-mode at B_T = 0.44 T and I_P = 0.9 MA
  - Scenario for reliable n = 1 excitation [1]
  - Neutral beam power is stepped down intentionally [1]
- Neutron rate is measured using F/G scintillator [2]
  - Simulated neutron rate using Kick model agrees better [3]
  - Experimental measurement is used as-is
  - Agreement is better considering measurement uncertainty
- Validated kick TRANSP is used for time dependent profiles



[1] La Haye *et al.*, Phys. Plasmas **19** 062506 (2012)
[2] Roquemore *et al.*, Proc. Symposium on Fusion Engineering SP1-39 (2011)
[3] Yang *et al.*, Plasma Phys. Control. Fusion **63** 045003 (2021)

Synthetic soft X-ray diagnostic reveals n = 1 mode structure

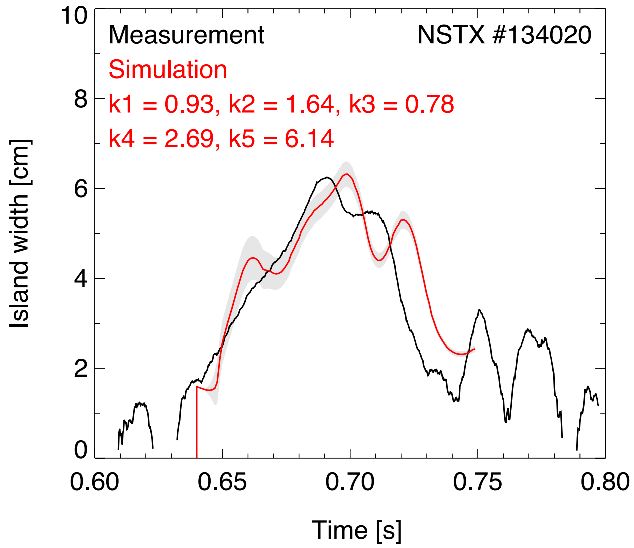
- Tomography of $\tilde{\varepsilon}_{SXR}$ is difficult
  - Forward-model line-integrated $\tilde{\varepsilon}_{SXR}$
  - Adjust mode parameters
  - Minimize difference vs. measured $\tilde{\varepsilon}_{SXR}$
- Structure of n = 1 system [1]
  - Core kink mode (nonresonant)
  - Magnetic island at q = 2 (i.e., m = 2)
- Input to kick TRANSP analysis



[1] Yang *et al*., Plasma Phys. Control. Fusion **63** 045003 (2021)

Simulated island width is compared with experiment

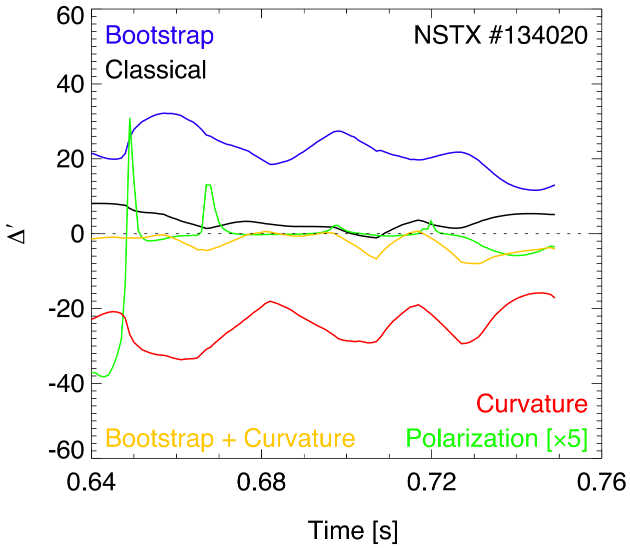
- Coefficients are adjusted to minimize difference
  - Fit result is a global minimum in optimization problem
- Onset of magnetic island is out of scope
  - Simulation starts at 0.64 s
  - Initial island width is set at 2 cm (3% of minor radius)
  - We are concerned on the factor that makes island grow
- Island grows like *w* ~ *t* (as would a classical TM)
  - Something needs to cancel out bootstrap current drive



$$\frac{1}{k_3}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_1\left[\frac{16J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2}\right] - k_2\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_4\frac{\beta_\theta\varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

Bootstrap drive is likely canceled out by curvature stabilization

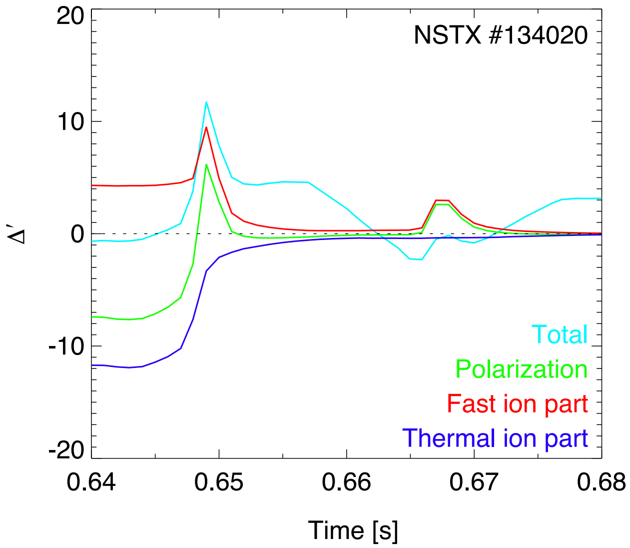
- Dimensionless stability indices (Δ') are compared
  - Curvature stabilization is large in spherical tori [1]
  - Balances bootstrap current drive
  - Helps simulate island growth like $w \sim t$
- Classical drive decreases with time
  - Need a push to maintain slope dw/dt
  - Can polarization current term provide a timely push?



$$\frac{1}{k_3}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_1\left[\frac{16J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2}\right] - k_2\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_4\frac{\beta_\theta\varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

Competing fast ion part in polarization current term is essential

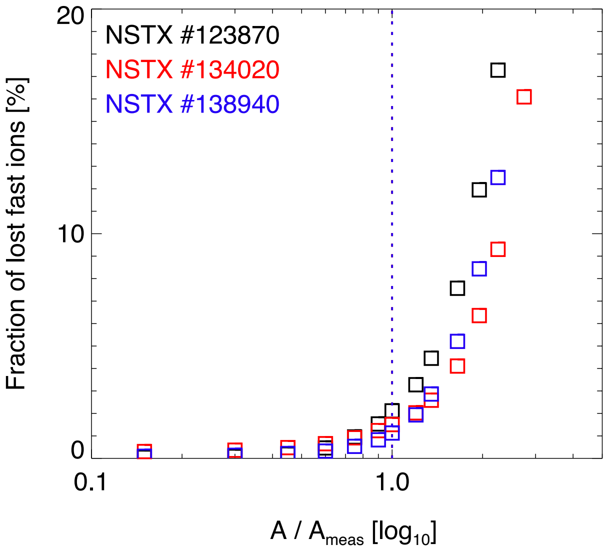
- Classical drive decreases with time
  - Fast ion part provides chance for $\Delta'_{\mathrm{pol}} > 0$
  - Simulation cannot follow measurement w/o fast ion part
- Spike in fast ion part is likely a numerical error
  - Ion density profile is flat near q = 2 surface
  - Sometimes $L_{n_i} \equiv n_i/\nabla n_i$ goes infinity as $\nabla n_i \to 0$
  - Fast ion transport is typically inside q = 2 surface
  - As a result, L_{n_h} does not go infinity, resulting in Δ'_{u} spike

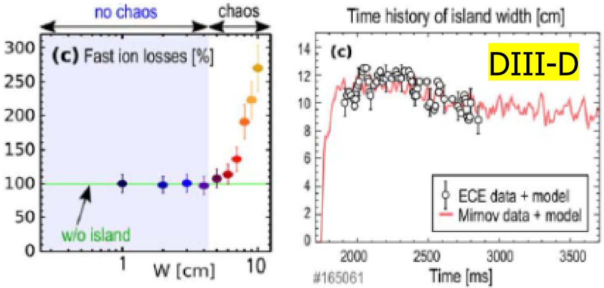


$$\frac{1}{k_3}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_1\left[\frac{16J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2}\right] - k_2\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_4\frac{\beta_\theta\varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

Island saturates at orbit stochasticization threshold in NSTX

- Three NSTX discharges have different *q* profiles [1]
  - As mode amplitude is scanned beyond measured…
  - Fast ion transport starts to increase rapidly
  - At DIII-D, threshold was at $A / A_{meas} \ll 1$ [2]



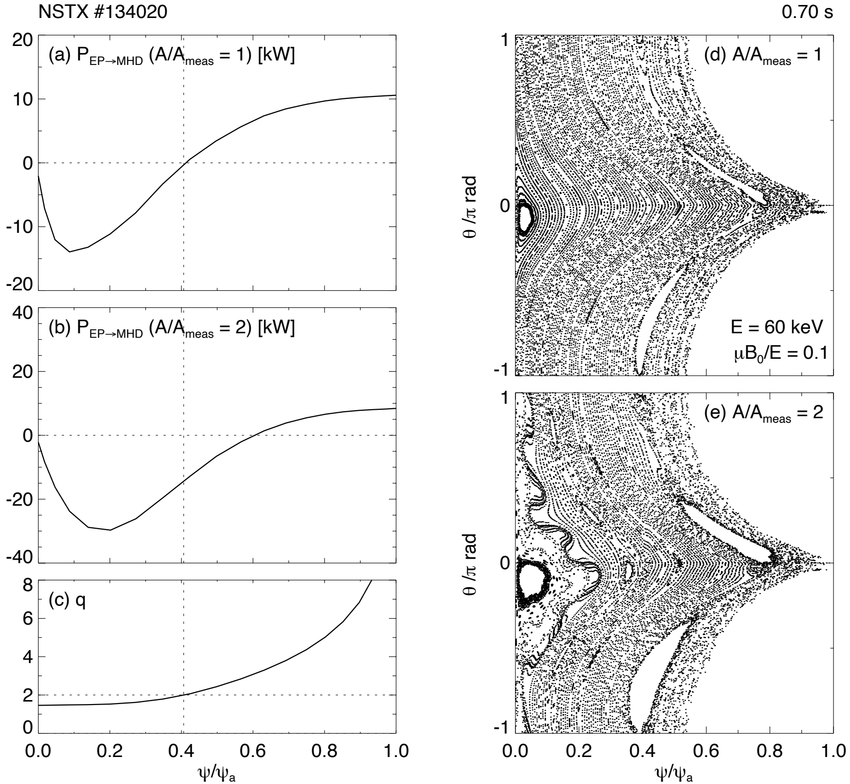


[1] Yang *et al*., Plasma Phys. Control. Fusion **63** 045003 (2021)
[2] Bardoczi *et al*., Plasma Phys. Control. Fusion **61** 055012 (2019)

Energy exchange profile shows fast ions taking energy from TM

- Stochastic transport is confirmed
  - Transport increases when…
  - KAM surfaces "start to" break [1]

- More fast ion transport for bigger island
  - Causes loss of passing fast ions
  - Less kinetic neoclassical polarization current
  - Island drive is reduced & growth saturates



[1] Collins *et al*., Phys. Rev. Lett. **116** 095001 (2016)

Passing fast ions may be essential for island growth in NSTX

- Passing fast ions is essential to opening the gate for island growth in NSTX #134020
  - Kinetic neoclassical polarization current provides valuable degree of freedom for NTM drive
  - Quantitative analysis of fast ion effect on NTM stability can be done using TRANSP
- Island growth saturation at orbit stochastiziation threshold is observed
  - Qualitative agreement with passing fast ion induced island drive theory
- Future work includes…
  - Further exploration into NSTX NTM database
  - Benchmark of classical Δ' calculation using STRIDE [1]
  - Comparison with DIII-D experiment result: Less fast ion contribution is expected in DIII-D

[1] Glasser and Koleman, Phys. Plasmas **25** 082502 (2018)

Questions remain due to uncertainty in island rotation direction

- Doppler shift makes measurement of island rotation frequency difficult
  - Island rotation at plasma frame is small, whereas Doppler shift (noise) is large [1]
  - Islands may change directions by turbulence [2]

- Assuming island rotates in ion diamagnetic direction ($\omega' < 0$):
  - Polarization current is stabilizing, giving rise to explanation to observed island saturation [3]
  - Kinetic neoclassical polarization current is destabilizing [4]
  - However, this contradicts previous assessment [5]

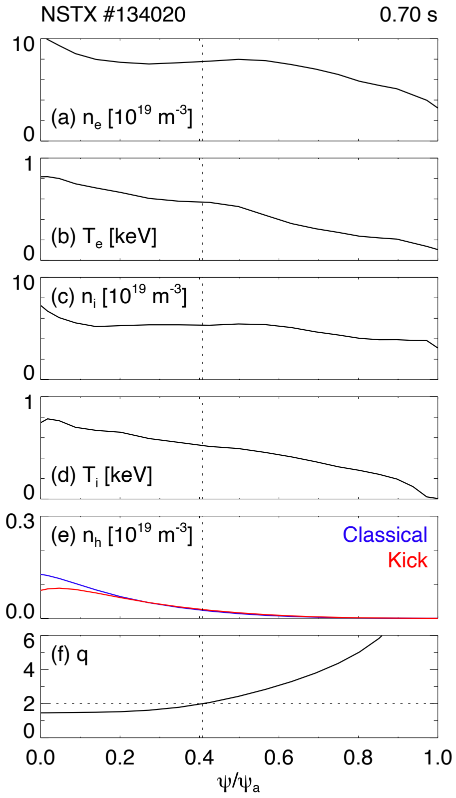
$$\Delta'_{pol} = -\epsilon^{3/2}\left(\frac{L_q}{L_p}\right)\frac{\rho_{\theta i}^2}{w^2}\frac{\beta_\theta}{w}\frac{\omega'(\omega' - \omega_{*i})}{\omega_{*e}^2}$$

$$\Delta'_{kin.nc-pol} = -\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)\frac{L_{n_i}}{L_h}\frac{n_h}{n}\frac{\omega'}{\omega_{*i}}$$

[1] La Haye *et al*., Phys. Plasmas **10** 3644 (2003)
[2] Hornsby *et al*., Plasma Phys. Control. Fusion **58** 014028 (2016)
[3] Wilson *et al*., Phys. Plasmas **3** 248 (1996)
[4] Cai Nucl. Fusion **56** 126016 (2016)
[5] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)

TRANSP is used to calculate profiles used for this study

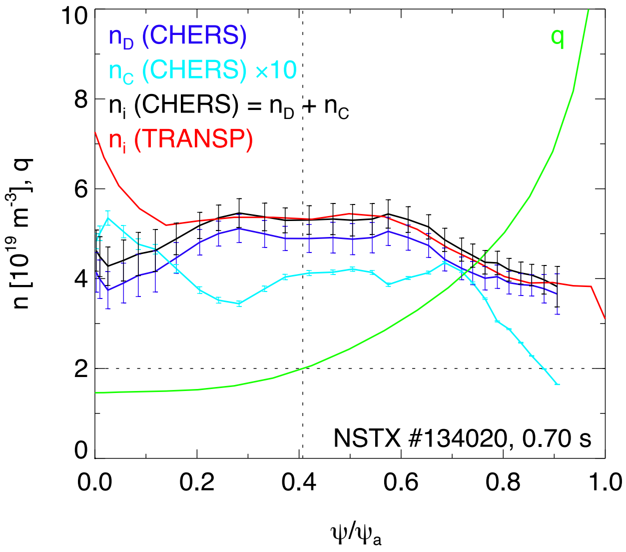
- NSTX #134020 is selected for analysis
  - Neutral beam heated H-mode at B_T = 0.44 T and I_P = 0.9 MA
  - Scenario for reliable n = 1 excitation [1]
- TRANSP is used for profiles
  - MSE [2] constrained equilibrium [3]: q = 2 at ψ_N = 0.4
  - Fast ion density [4]: MHD induced transport captured at ψ_N < 0.4



[1] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)
[2] Levinton and Yuh, Rev. Sci. Instrum. **79** 10F522 (2008)
[3] Menard *et al*., Phys. Rev. Lett. **97** 095002 (2006)
[4] Podestà *et al*., Plasma Phys. Control. Fusion **56** 055063 (2014)

Kinetic neoclassical polarization current term introduces $n_i(\psi_N)$

- Fast ions make ion density important for NTM
- CHERS provides ion density profile [1]
  - Measures carbon density profile
  - Additional input of Z_{eff} is needed for ion density profile
  - Ion density profile is needed for Z_{eff} profile
- TRANSP validates measured n_i near $q = 2$
  - Core disagreement is likely due to carbon accumulation
  - TRANSP considers CHERS + TS: Likely more accurate



[1] Podestà *et al.*, Rev. Sci. Instrum. **79** 10E521 (2008)

$$\frac{1}{k_r}\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + k_b\left[\frac{16J_{BS}}{s\langle J\rangle}\frac{w}{w^2+w_d^2}\right] - k_p\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2 - k_c\frac{\beta_\theta\varepsilon^2}{rw}\frac{L_q^2}{|L_p|}\frac{q^2-1}{q^2}$$

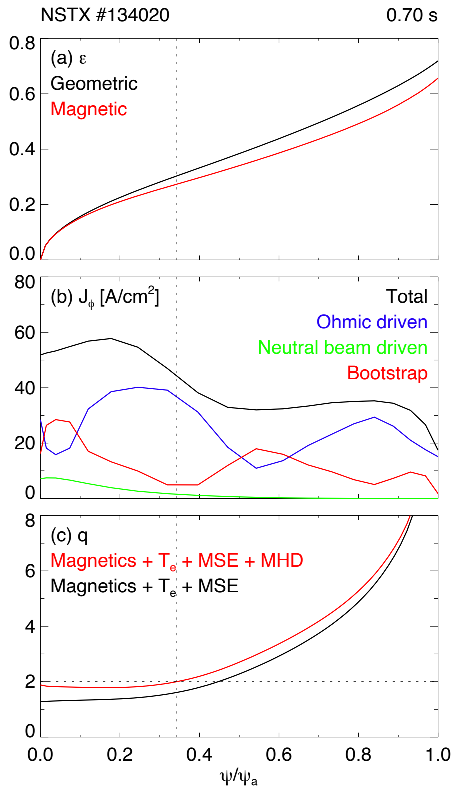
Special considerations are made for the simulation

- Geometric and magnetic ε are different in spherical tori
  - Rigorously, toroidal effects come from magnetic ε_B [1]

$$\varepsilon_B \equiv \frac{B_{in} - B_{out}}{B_{in} + B_{out}}$$

- Bootstrap current is calculated from NCLASS model [2]
  - Evolution of n_e and/or T_e is not the same as that of β_θ [3]

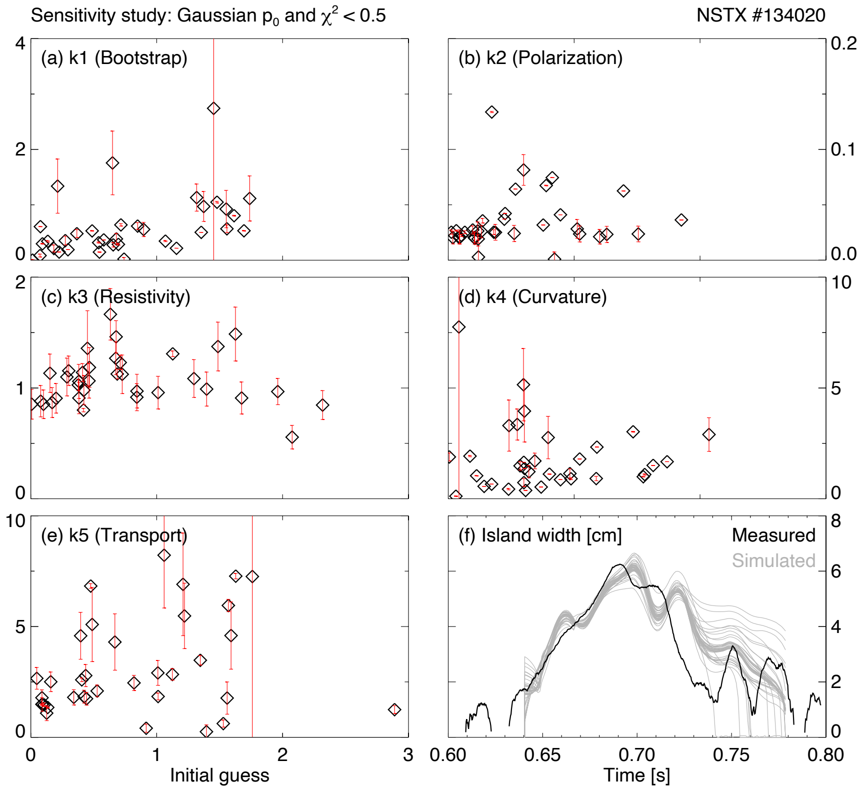
- Island location is used as extra constraint for q



[1] La Haye *et al.*, Phys. Plasmas **19** 062506 (2012)
[2] Houlberg *et al.*, Phys. Plasmas **4** 3230 (1997)
[3] Fredrickson *et al.*, Phys. Plasmas **7** 4112 (2000)

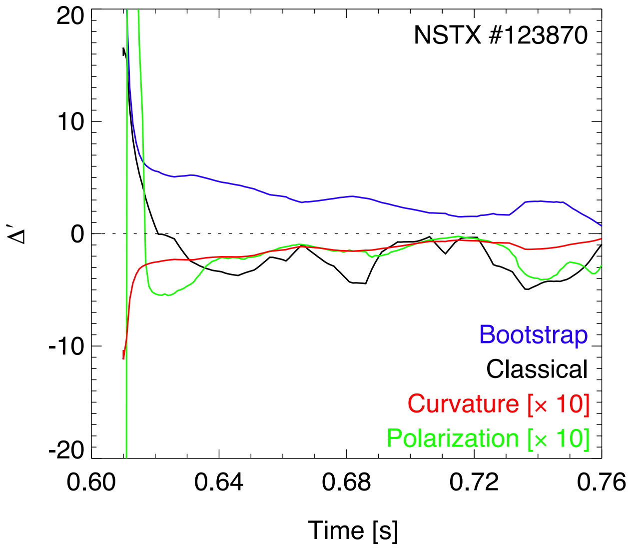
Fit result is a global minimum in optimization problem

- Optimization problem can converge to…
  - One of the local minima
  - Global minimum
- If the solution is a global minimum…
  - Fit result would be insensitive to initial guess
  - Box scatter in result vs. input graph
  - All coefficients show box scatter
- Coefficient k5 has large uncertainty
  - Reasonable
  - Involves cross field diffusion term



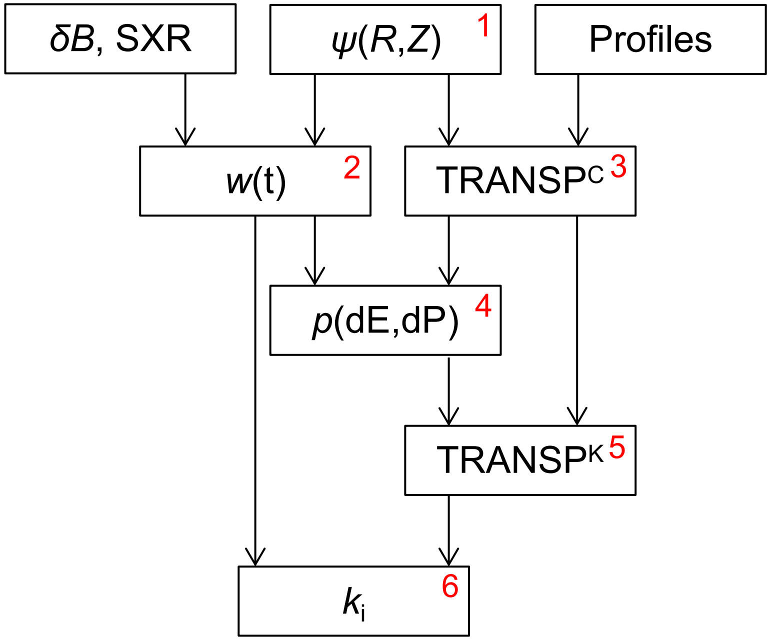
Some physics are missing in the model

- Parallel current effect when island grows larger
  - Poloidal Larmor radius for beam ions in NSTX ≤ 15 cm
- Different sources of bootstrap current [1]
- Effect of island rotation
- In NSTX #123870, classical *Δ′* drops negative
  - Need more freedom in driving terms
  - At low magnetic shear, trapped fast ions may affect classical *Δ′* [2]



[1] Gorelenkov *et al.*, Phys. Plasmas **3** 3379 (1996)
[2] Halfmoon and Brennan, Phys. Plasmas **24** 062501 (2017)

NTM-GRE analysis procedure has been developed

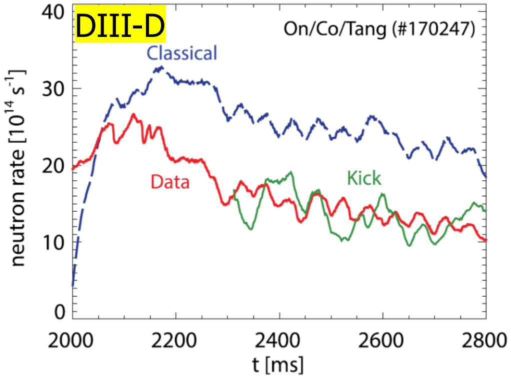


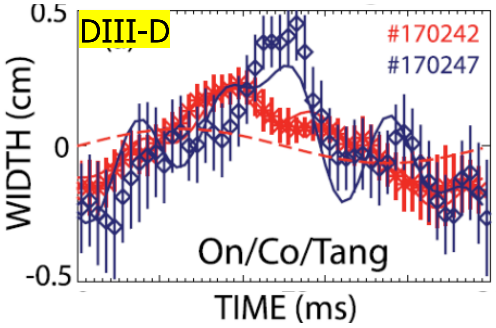
1. Run EFIT
2. Run NTM-SXR
   – Determine [tnorm] and [tscale]
3. Run TRANSP (classical)
4. Run ORBIT
5. Run TRANSP (kick)
   – Determine go/rerun based on [Sn]
6. Run NTM-GRE

- Measured: δB, SXR, and other profiles
- Total 6 steps and 2 decision points

Fast ions interact with NTM as they do with AE

- Fast ions interact with Alfvén eigenmodes (AEs) [1]
- Fast ions interact with neoclassical tearing modes (NTMs)
  - NTMs cause fast ion transport
  - Model validated qualitatively [2] and quantitatively [3]
  - NTM chirp is correlated with fast ion activity [4]
  - Model validation inconclusive [3]
- Use TRANSP to study fast ion effect on NTM stability





[1] Podestà *et al*., Plasma Phys. Control. Fusion **59** 095008 (2017)
[2] Zweben *et al*., Nucl. Fusion **39** 1097 (1999)
[3] Heidbrink *et al*., Nucl. Fusion **58** 082027 (2018)
[4] Fredrickson, Phys. Plasmas **9** 548 (2002)

Island width is calculated from Mirnov coil, EFIT, and SXR data

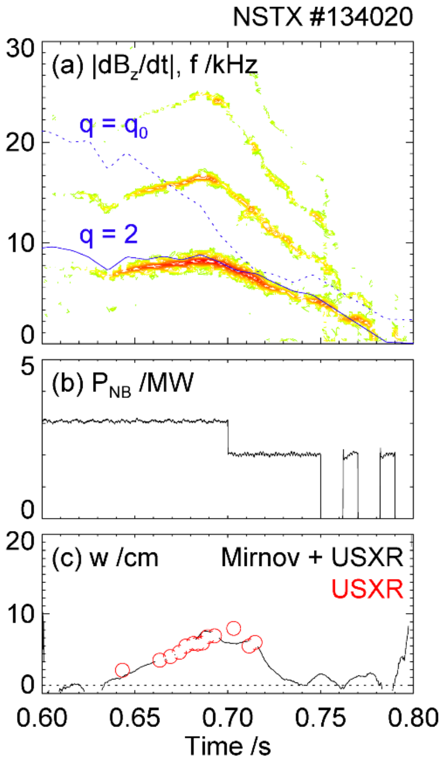
- TRANSP provides time-dependent solution
  - Time-dependent input is provided for NTM parameters

- Island width time evolution $w(t)$ is [1]:

$$w^2 = g(rb_r q/mB_\theta q')$$

where it relates Mirnov signal by $b_r \approx (1/2)(r_w/r)^{m+1} b_\theta$ [2]

  - From linear, cylindrical, ideal, low-β tearing mode equation
  - Constant g accounts for simplifications
    - Determined by scaling to synthetic SXR diagnostics [3]



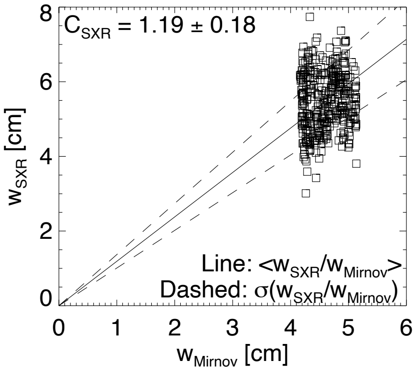
[1] Chang *et al.*, NF **34** 1309 (1994)
[2] La Haye *et al.*, PoP **7** 3349 (2000)
[3] Yang *et al.*, Plasma Phys. Control. Fusion **63** 045003 (2021)

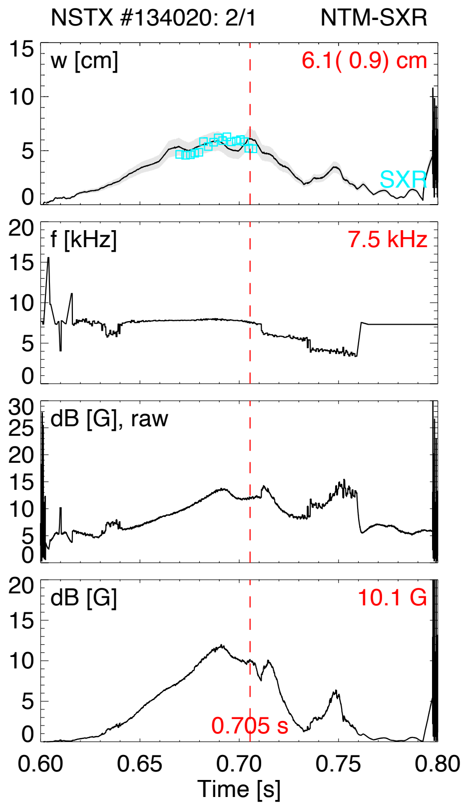
Magnetic island width and its error bar are determined from SXR

- Island width is related to Mirnov coil signal

$$w = Cw_{Mirnov}$$

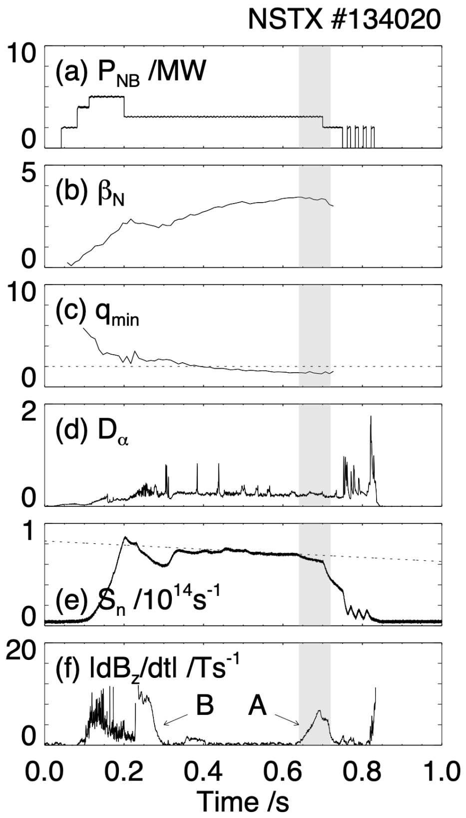
  - Assuming SXR points are true, $C = \langle w_{SXR}/w_{Mirov} \rangle$
  - Standard deviation determines error $\delta C = \sigma(w_{SXR}/w_{Mirov})$
  - Error bar is therefore $\delta w = 2\sigma w_{Mirnov}$
  - Take maximum error bar as a representative value





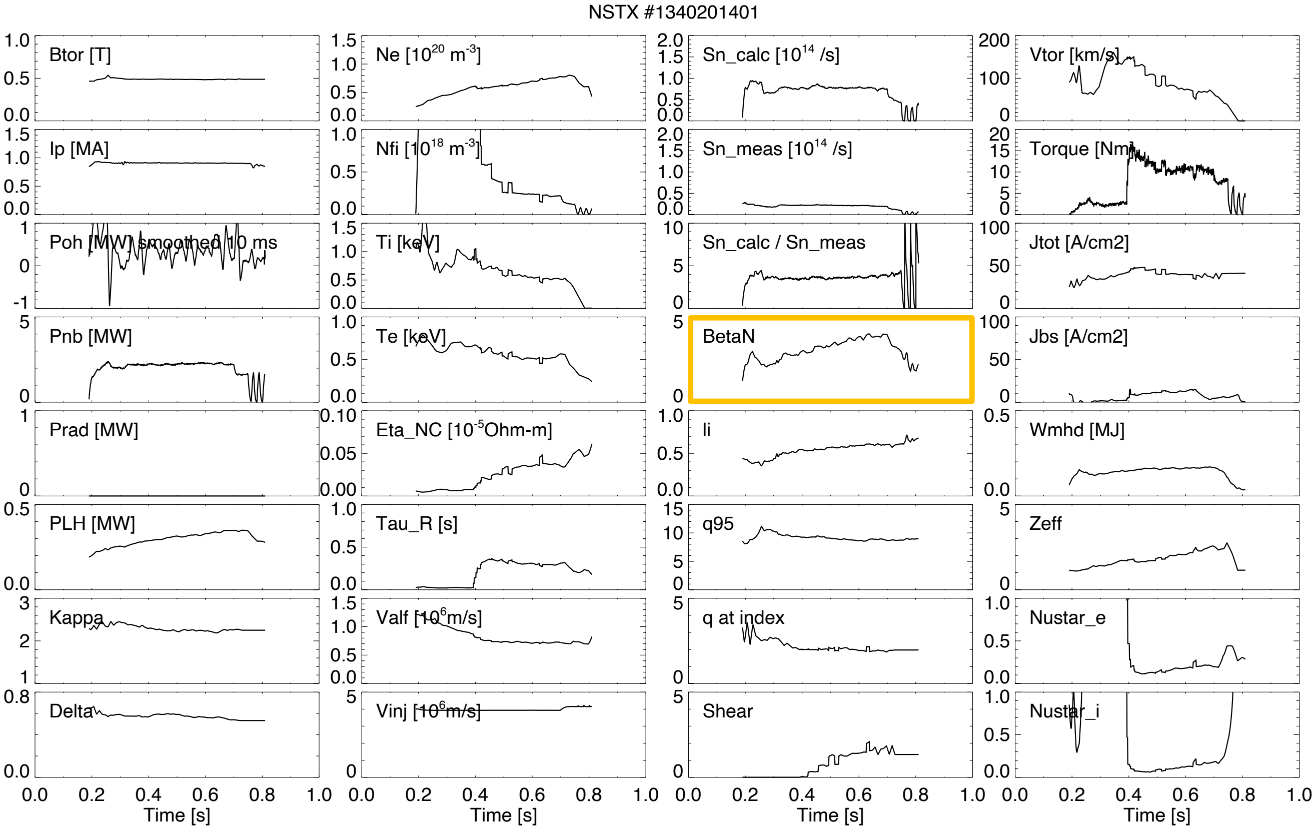
What is triggering the NTM?

- NSTX is transient – β_N is still rising
  - ELM free
  - Sawteeth free ($q_{min} > 1$)
  - The "spontaneous" NTM seen in TFTR [1]



[1] Fredrickson, Phys. Plasmas **9** 548 (2002)

Beta N growth might have "triggered" the NTM (triggerless NTM)



Does island grow like classical tearing mode in NSTX?

- Consider modified Rutherford equation

$$\frac{\tau_R}{r^2}\frac{dw}{dt} = \Delta'_{m,n}(w) + \frac{16J_{BS}}{s\langle J\rangle}\frac{1}{w}$$

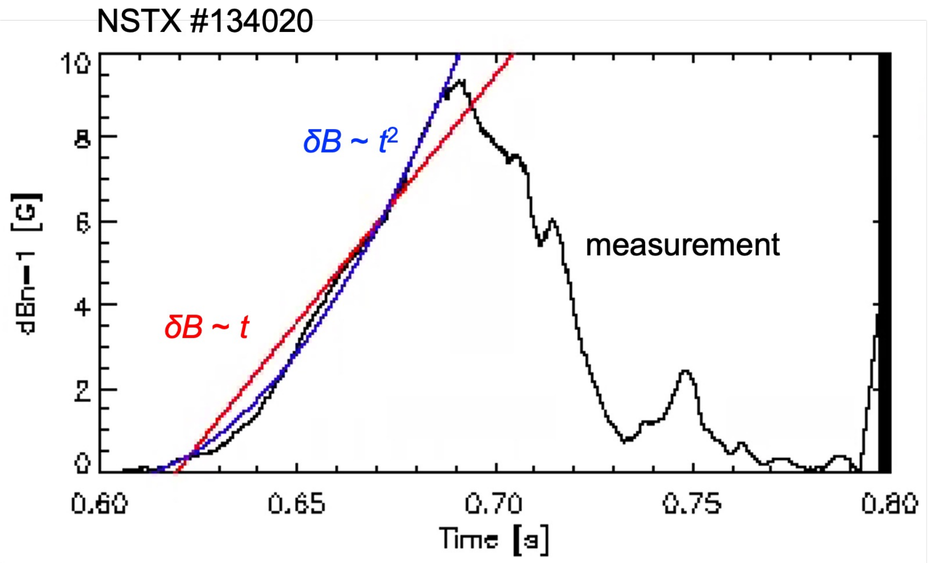
  - When classical term dominates

$$w \sim Ct$$
$$\delta B \sim Ct^2$$

  - When bootstrap current term dominates

$$w\, dw/dt \sim N$$
$$w^2 \sim Nt$$
$$\delta B \sim Nt$$

- Consider $w \sim \sqrt{\delta B}$ for the last lines



On calculation of classical delta prime

- Tearing mode equation is solved for helical flux functions
  - Considered island width dependence [1]
  - Considered reversed shear plasmas [2]
  - Considered interaction with walls [3]

[1] White *et al*., Phys. Fluids **20** 800 (1977)
[2] Fredrickson *et al*., Phys. Plasmas **7** 4112 (2000)
[3] Nave and Wesson, Nucl. Fusion **30** 2575 (1990)

Previous assessment of island frequency in plasma frame [1]

TABLE I. Evaluation of sources of small island effects at the marginal point for m/n = 2/1.

	NSTX #134020	DIII-D #133577	DIII-D #135861
$\varepsilon^{1/2}$	0.567	0.478	0.402
w_{marg} (cm)	3.42	2.76	1.71
$w_{bi} = \varepsilon^{1/2}\rho_{\theta i}$	1.40	0.89	0.99
w_d (cm)	1.07	1.03	0.82
$(\nu_i/\varepsilon)/\lvert\omega_e^*\rvert$	0.84	2.73	1.09
ω/ω_i^*	0.23 ± 0.37	−0.20 ± 0.38	−0.07 ± 0.30
$(3L_q/L_{pe})^{1/2}w_{bi}$	3.01	1.52 (4.60)[a]	2.16

[a]High collisionality is enhanced by $\varepsilon^{-3/4}$.

[1] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)

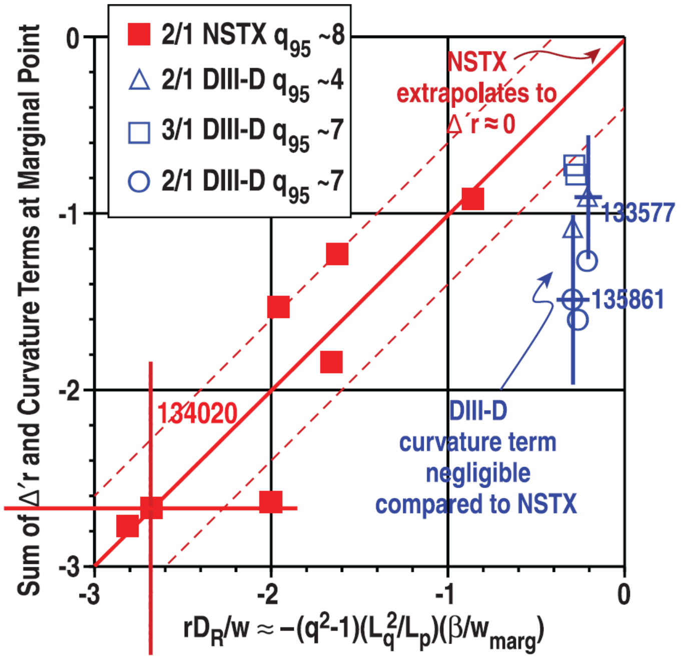
On curvature term in spherical tori

- Consider that relative strength D_R/D_{NC} is like $f(\varepsilon)$
  - Smaller for DIII-D but significant for NSTX
  - Assuming $q \approx \varepsilon(B_T/B_\theta)\,(1+\kappa^2)/2$
  - Curvature term must be included for ST [1]

$$k_3 \frac{\tau_R}{r}\frac{dw}{dt} = \left[\Delta' + \frac{rD_R}{w} + \frac{rD_{NC}}{w}\right] r$$

- Standard approximation works: See figure

$$D_R \approx -(q^2-1)\left(L_q^2/rL_p\right)\beta$$

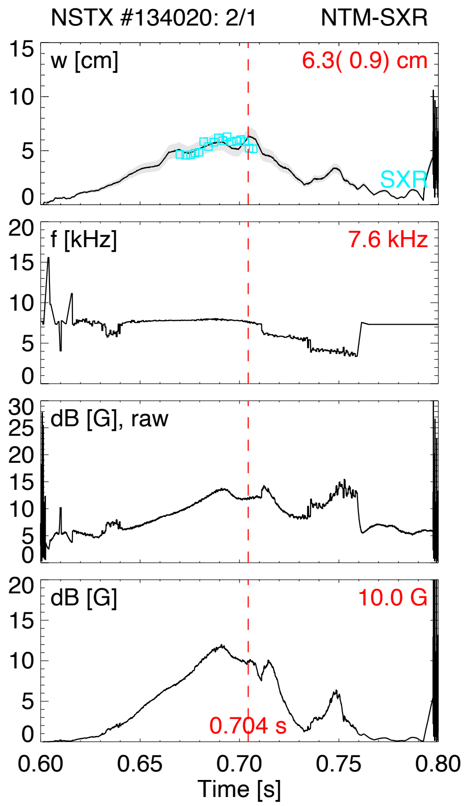


[1] La Haye *et al*., Phys. Plasmas **19** 062506 (2012)

Reproduced from Figure 8 [1]

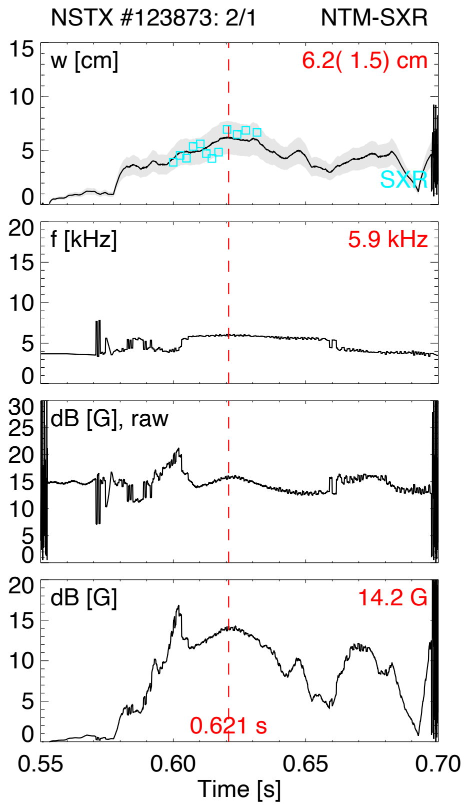
NSTX #134020

- H-mode with reliable rotating n = 1 onset
  - I_P = 0.9 MA, B_T = 0.44 T
  - P_{NB} steps down from 3 to 2 MW at 0.7 s
  - Lithiumization and n = 1 and n = 3 error field correction
- SXR/Mirnov conversion factor is C_{SXR} = 1.17 ± 0.16
- Rotating n = 1 saturates at 6.3 cm
  - Onset at around 0.63 s when δB = 1 G
  - Peak at 0.704 s when δB = 10 G
  - Rotation is steady at 7.6 kHz (q = 2 from LRDFIT06)



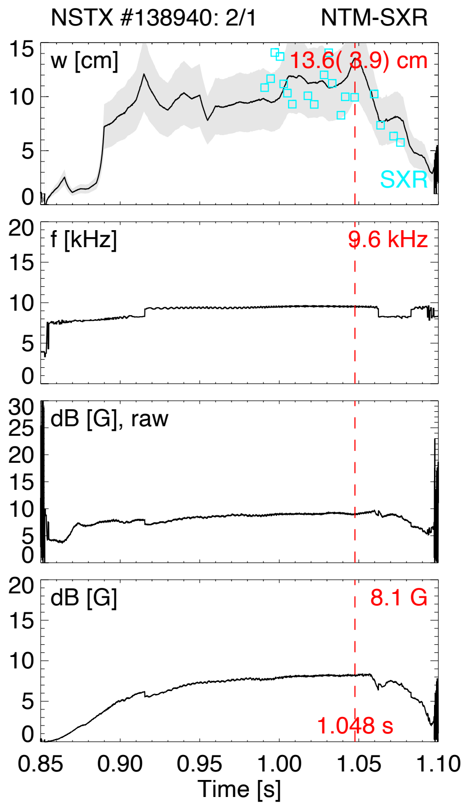
NSTX #123873

- Lower rotation
  - I_P = 1.0 MA, B_T = 0.44 T
  - P_{NB} steps down from 4 to 2 MW at 0.6 s
- SXR/Mirnov conversion factor is C_{SXR} = 1.91 ± 0.47
- Rotating n = 1 saturates at 6.2 cm
  - Onset at around 0.58 s when δB = 1 G
  - Peak at 0.621 s when δB = 14.2 G
  - Rotation is steady at 5.9 kHz (q = 2 from LRDFIT06)

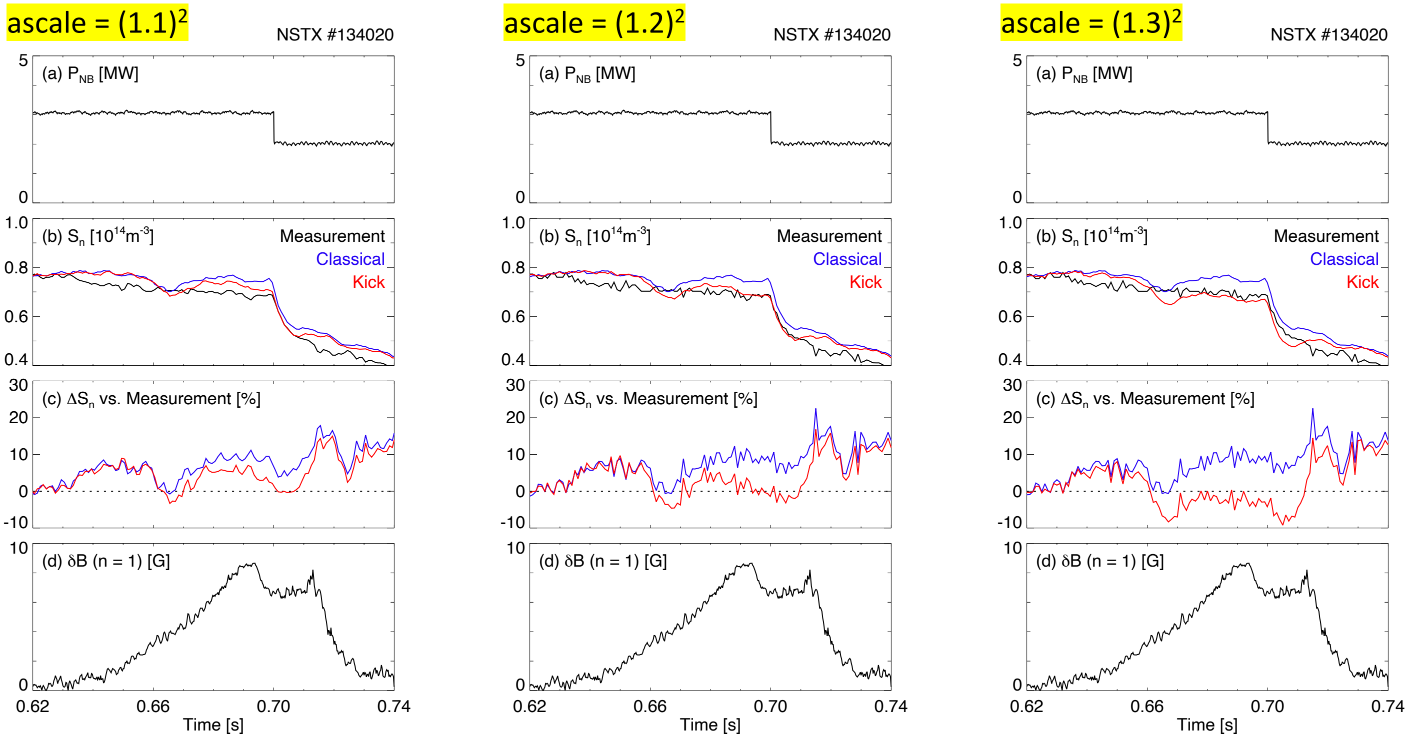


NSTX #138940

- High-triangularity, low-elongation
  - I_P = 0.8 MA, B_T = 0.44 T
  - P_{NB} = 4 MW is modulated by β_N controller
- SXR/Mirnov conversion factor is C_{SXR} = 1.25 ± 0.36
- Rotating n = 1 saturates at 13.6 cm
  - Onset at around 0.87 s when δB = 1 G
  - Peak at 1.048 s when δB = 8.1 G
  - Rotation is steady at 9.6 kHz (q = 2 from LRDFIT06)



Mode amplitude may have been underestimated by 20%



Measured island width is 6.3 cm, 20% is 1.2 cm (smaller than SXR resolution)